Introduction to Fluid Mechanics 1 st Midterm Exam **(sample exam)** 13.03.2018

Time for the exam: 8:15 to 10:00.

Name:

NOTE: In multiple-choice questions, choose ONLY one answer.

Questions

Q1. () What are the units corresponding to the following variables (in SI)?

ρ (density): kg/m^3	μ (dynamic viscosity): $N.s/m^2$
P (Pressure): N/m^2 or Pa	τ (shear stress): N/m^2
Ev (Bulk modulus of Elasticity): N/m^2 or Pa	S (specific gravity): dimensionless

Q2. () In the lower part of the stratosphere, the temperature:

a) Increases exponentially with height
b) Increases logarithmically with height
c) Increases linearly with height
d) <u>Is constant with height</u>
e) Decreases linearly with height
f) Decreases logarithmically with height
g) Decreases exponentially with height
h) None of the above

And the pressure:

a) Increases with height
b) Is constant
c) <u>Decreases with height</u>
d) None of the above

Q3. () When streamlines are curved, pressure will --**increase**------------ outward from the center of the curvature.

a) Increase

b) Decrease
c) Either increase or decrease

Q4. () Pathlines, streaklines and streamlines are coincident in ---**steady**--------- flows.

a) Potential
b) Steady
c) Unsteady
d) None of them

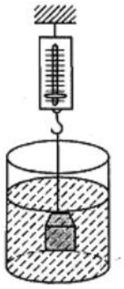
Q5. () What can one measure with a water column barometer? (Really short answer, please) Do we need to calibrate such an instrument? (Yes/No)

A5. One can measure the atmospheric pressure.
We do not need to calibrate the instrument, since it is simply based on the hydrostatic equation.

Q6. () What height would a water barometer need to be to measure atmospheric pressure? (ρ_{water}=1000 kg/m^3, and $p_{atmosphere}$=100 kPa)

$$p_{atmosphere} \approx 1bar = 1 \times 10^5\ N/m^2$$
$$10^5 = \rho g h$$
$$h = \frac{10^5}{1000 \times 9.81} = 10.19\, m\ of\ water$$
$$h = \frac{10^5}{(13.6 \times 10^3) \times 9.81} = 0.75 m\ of\ mercury$$

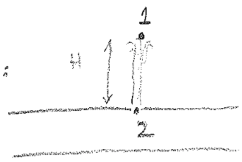
Q7. () The figure shows an object of mass 0.4 kg and volume 2.0 x 10^{-4} m^3 that is suspended from a scale and submerged in a liquid. If the reading on the scale is 3 N, then what the buoyant force that the fluid exerts on the object is? What is the density of the liquid?



$$W - T - F_b = 0;$$
$$F_b = 0.4 \times 9.81 - 3 = 0.924\ N$$
$$F_b = \rho_L g V \rightarrow \rho_L = \frac{0.924}{9.81 \times 2.0 \times 10^{-4}} = 470.95\ kg/m^3$$

Q8. () The pressure in domestic water pipes is typically 500 KPa above atmosphere. If viscous effects are neglected, determine the height reached by the jet of water through a small hole in the top of the pipe. (Density of water=1000 kg/m^3, g=10 m/s^2)

• from Bernoulli equation between points 1 and 2:



$$\underbrace{P_1}_{P_{atm}} + \frac{1}{2}\rho \underbrace{V_1^2}_{0} + \rho g z_1 = P_2 + \frac{1}{2}\rho \underbrace{V_2^2}_{0} + \rho g z_2$$

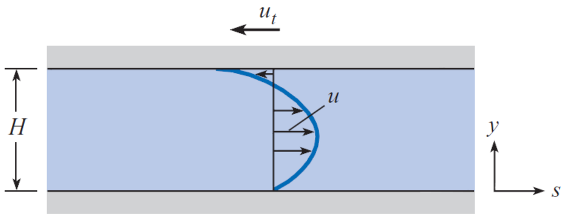
$$\Rightarrow \rho g (z_1 - z_2) = \rho g H = P_2 \Rightarrow H = \frac{P_2}{\rho g} = \frac{500 \times 10^3 \text{ Pa gage}}{1000 \frac{kg}{m^3} \times 10 \frac{m}{s^2}}$$

$$= \boxed{50 \text{ m}}$$

Problems

P1. () A laminar flow occurs between two horizontal parallel plates under a pressure gradient *dp/ds* (*p* decreases in the positive s direction). The upper plate moves left (negative) at velocity u_t. The expression for local velocity u is given as:

$$u = -\frac{1}{2\mu}\frac{dp}{ds}(Hy - y^2) + u_t\frac{y}{H}$$



a. Is the magnitude of the shear stress greater at the moving plate (y=H) or at the stationary plate (y=0)?
b. Derive an expression for the y position of zero shear stress.
c. Derive an expression for the plate speed u_t required to make the shear stress zero at y=0.

PLAN

By inspection, the rate of strain (du/dy) or slope of the velocity profile is larger at the moving plate. Thus, we expect shear stress τ to be larger at $y = H$. To check this idea, find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$. Evaluate and compare the shear stress at the locations $y = H$ and $y = 0$.

SOLUTION

Part (a)

1. Shear stress, from definition of viscosity

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \mu\frac{d}{dy}\left[-\frac{1}{2\mu}\frac{dp}{ds}\left(Hy - y^2\right) + u_t\frac{y}{H}\right] \\
&= \mu\left[-\frac{H}{2\mu}\frac{dp}{ds} + \frac{y}{\mu}\frac{dp}{ds} + \frac{u_t}{H}\right] \\
\tau\,(y) &= -\frac{(H - 2y)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H}
\end{aligned}$$

Shear stress at $y = H$

$$\begin{aligned}\tau(y=H) &= -\frac{(H-2H)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H} \\ &= \frac{H}{2}\left(\frac{dp}{ds}\right) + \frac{\mu u_t}{H} \qquad (1)\end{aligned}$$

2. Shear stress at $y = 0$

$$\begin{aligned}\tau(y=0) &= -\frac{(H-0)}{2}\frac{dp}{ds} + \frac{\mu u_t}{H} \\ &= -\frac{H}{2}\left(\frac{dp}{ds}\right) + \frac{\mu u_t}{H} \qquad (2)\end{aligned}$$

Since pressure decreases with distance, the pressure gradient dp/ds is negative. Since the upper wall moves to the left, u_t is negative. Thus, maximum shear stress occurs at $y = H$ because both terms in Eq. (1) have the same sign (they are both negative.) In other words,

$$|\tau(y=H)| > |\tau(y=0)|$$

.

$$\boxed{\text{Maximum shear stress occur at } y = H}.$$

Part (b)
Use definition of viscosity to find the location (y) of zero shear stress

$$\begin{aligned}\tau &= \mu\frac{du}{dy} \\ &= -\mu(1/2\mu)\frac{dp}{ds}(H-2y) + \frac{u_t\mu}{H} \\ &= -(1/2)\frac{dp}{ds}(H-2y) + \frac{u_t\mu}{H}\end{aligned}$$

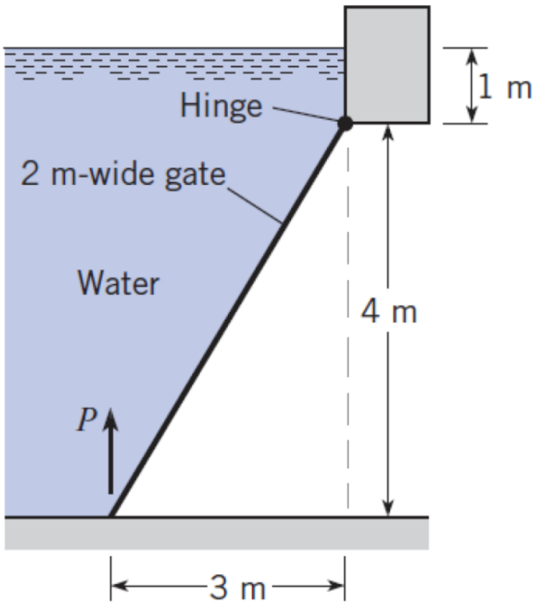
Set $\tau = 0$ and solve for y

$$\begin{aligned}0 &= -(1/2)\frac{dp}{ds}(H-2y) + \frac{u_t\mu}{H} \\ &\boxed{y = \frac{H}{2} - \frac{\mu u_t}{H dp/ds}}\end{aligned}$$

Part (c)

$$
\begin{aligned}
\tau &= \mu\frac{du}{dy} = 0 \text{ at } y = 0 \\
\frac{du}{dy} &= -(1/2\mu)\frac{dp}{ds}(H - 2y) + \frac{u_t}{H} \\
\text{Then, at } y &= 0 : du/dy = 0 = -(1/2\mu)\frac{dp}{ds}H + \frac{u_t}{H} \\
\text{Solve for } u_t &: \boxed{u_t = (1/2\mu)\frac{dp}{ds}H^2} \\
\textit{Note} &: \text{because } \frac{dp}{ds} < 0, u_t < 0.
\end{aligned}
$$

P2. () Determine P necessary to just start opening the 2 m–wide gate.



A2.

SOLUTION

The length of gate is $\sqrt{4^2 + 3^2} = 5$ m

Hydrostatic force

$$\begin{aligned} F &= \bar{p}A \\ &= (\gamma \Delta z)\, A \\ &= (9810\,\mathrm{N}\,/\,\mathrm{m}^3)(3\,\mathrm{m})(2\,\mathrm{m} \times 5\,\mathrm{m}) \\ &= 294.3 \text{ kN} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{((2 \times 5^3)\,/12)\,\mathrm{m}^4}{(2.5\,\mathrm{m} + 1.25\,\mathrm{m})\,(2\,\mathrm{m} \times 5\,\mathrm{m})} \\ &= 0.555\,6\,\mathrm{m} \end{aligned}$$

Equilibrium

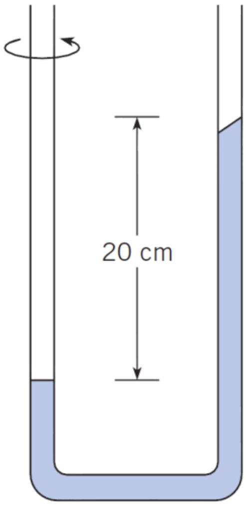
$$\sum M_{\text{hinge}} = 0$$

$$294.3\,\mathrm{kN} \times (2.5\,\mathrm{m} + 0.555\,6\,\mathrm{m}) - (3\,\mathrm{m})\,P = 0$$

$$P = 299.\,75\,\mathrm{kN}$$

$$\boxed{P = 300\,\mathrm{kN}}$$

P3. () A manometer is rotated around one leg, as shown. The difference in elevation between the liquid surfaces in the legs is 20 cm. The radius of the rotating arm is 10 cm. The liquid in the manometer is oil with a specific gravity of 0.8. Find the number of g's acceleration in the leg with greatest amount of oil.



Situation:

A manometer is rotated about one leg.

$\Delta z = 20\,\text{cm}$, $r = 10\,\text{cm}$, $S = 0.8$.

Find:

Acceleration in g's in leg with greatest amount of oil.

PLAN

Apply the pressure variation equation for rotating flow between the liquid surfaces of 1 & 2Let leg 1 be the leg on the axis of rotation. Let leg 2 be the other leg of the manometer.

SOLUTION

Pressure variation equation- rotating flow

$$\begin{aligned}
p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} &= p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2} \\
0 + \gamma z_1 - 0 &= \gamma z_2 - \frac{\gamma}{g} \frac{r_2^2 \omega^2}{2} \\
\frac{r_2^2 \omega^2}{2g} &= z_2 - z_1 \\
a_n &= r\omega^2 \\
&= \frac{(z_2 - z_1) 2g}{r_2} \\
&= \frac{(0.20)(2\text{g})}{0.1} \\
& \boxed{a_n = 4g}
\end{aligned}$$

P4. () In a contraction pipe, the centerline velocity is varying with time, t and distance inside the contraction, x, as follows:

$$V(x,t) = \frac{U_0 t / t_0}{(1-0.5x/L)^2}; \qquad L = 2m, \quad U_0 = 2m/s, \quad t_0 = 1s$$

For time t = 3s and x = 0.5 L:

(a) What is the local acceleration at the contraction?

(b) What is the convective acceleration at the contraction?

(a) local acceleration: ① $a_\ell = \frac{dV}{dt} = \frac{U_0/t_0}{(1-0.5X/L)^2}$

at $t = 3s$ and $X = 0.5L$,

② $a_\ell = \frac{2}{(1-0.5(0.5L)/L)^2} = \frac{2}{(3/4)^2} = \frac{32}{9} =$ **3.55 m/s²**

(b) convective acceleration: ① $a_c = V\frac{dV}{dX}$

$$a_c = \frac{(U_0/t_0)t}{(1-0.5X/L)^2}\left[\frac{-2(U_0/t_0)\,t}{(1-0.5X/L)^3}\left(\frac{-0.5}{L}\right)\right]$$

at $t = 3s$ and $X = 0.5L$:

② $$a_c = \frac{2(3)}{(1-0.5(0.5L)/L)^2}\left[\frac{2(2)(3)}{(1-0.5(0.5L)/L)^3}\left(\frac{0.5}{2}\right)\right]$$

$a_c = 75.85$ m/s²